Primer on Problem-Solving using Multiple Representations

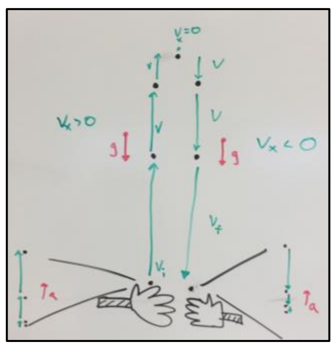
By Brian Frank (Middle Tennessee State University)

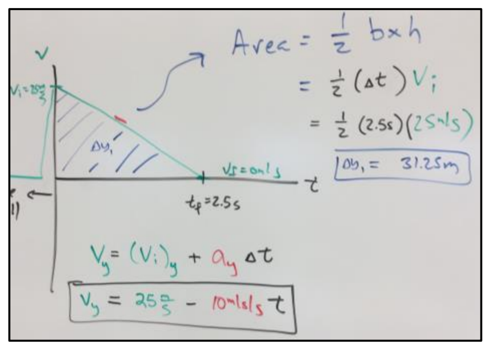
Throughout our algebra-based physics curriculum, there is an emphasis on students using **multiple representations**. For example, in the study of kinematics, this would include motion diagrams, tables, various graphs, and equations. During instruction, we try to start students off with some concrete experiences of the phenomena that are to be represented, and then gradually move from more concrete representations to more abstract ones. An example of moving from concrete to abstract for a tossed ball might look something like this.

Phenomena
- Observing a ball tossed in the air

Concrete Reps
- Motion Diagrams

Abstract Reps
- Graphs and their
- Algebraic Expressions





Students are required to make use of multiple representations like this in the process of *solving problems*. We ask them to do the work of explicitly drawing connections using multiple representations. As such, problem solving becomes more about mapping out important relationships in the problem than it is about moving through specific steps.

You might be wondering, "How is this different?" In part, I would say that a lot of curriculum <u>do</u> ask students to learn a variety of representations, but often students do not learn how to relate those representations to each other very well. Furthermore, using representations can often be disconnected from the practice of problem-solving. For example, in our previous curriculum we asked students to practice interpreting kinematics graphs, but our curriculum didn't do much work to integrate kinematics graphs as a tool for solving problems. Students were more likely to see graphing as related to a fun activity with a motion detector, a random question on a test, or something to worry about with labs.

Example of Solving Kinematics Problems with Multiple Representations:
It's helpful to see examples of students using multiple representations to approach a problem. Two pictures below show student solutions to a problem of determining the distance needed to come to a stop, given an initial speed, reaction time, and rate of acceleration.

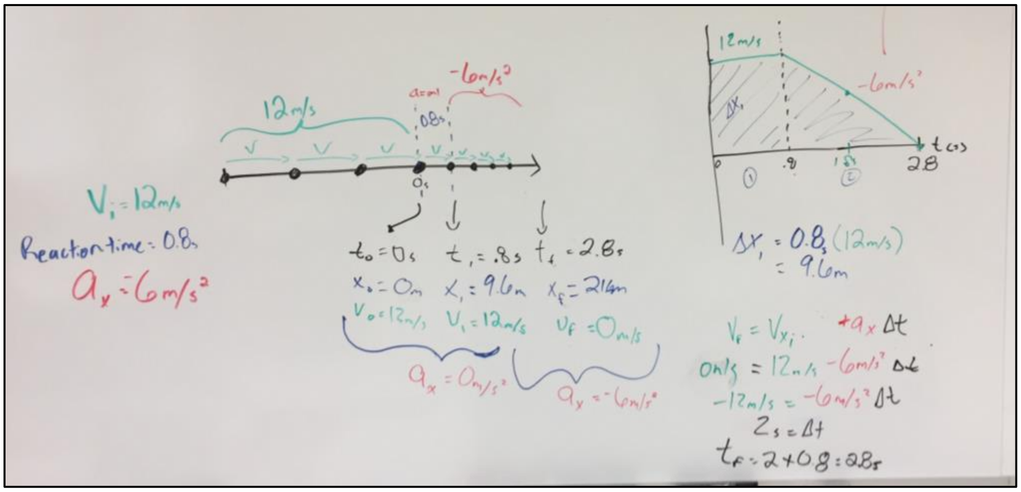
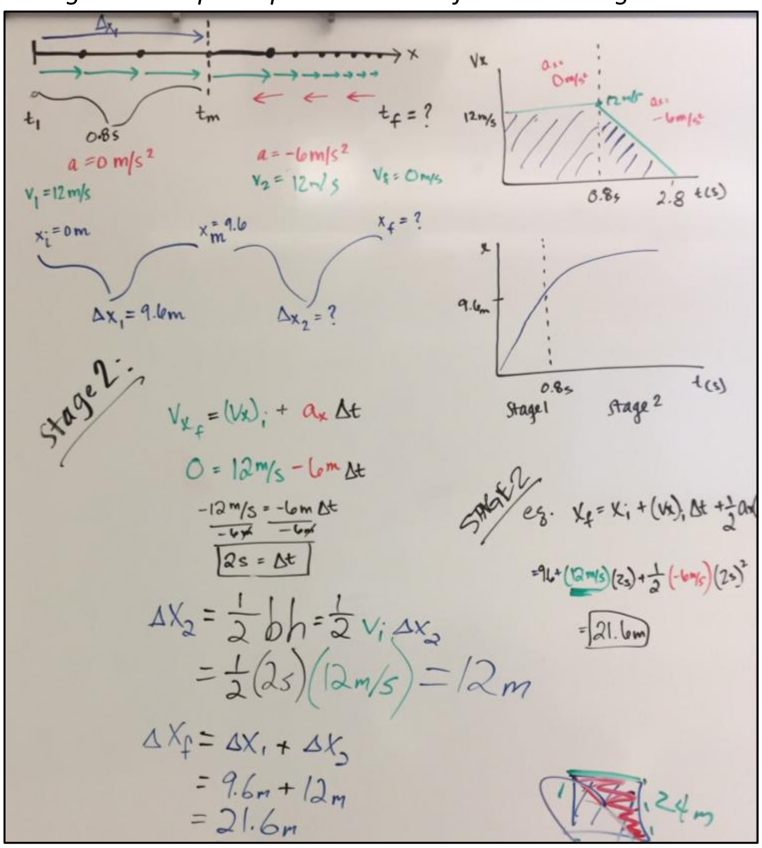


Figure: Multiple Representations of a Car Braking



Both of the solutions above show students using motion diagrams, graphs, and equations. These examples are not meant to be perfect, but rather representative of the kind of work you can expect students to be able to do.

I think it's also helpful to compare and contrast these solutions to what we did before.

- In our previous curriculum, students were asked to approach kinematics problems by (i) first making a pictorial representation to help visualize, (ii) then generating a list of quantitative variables and (iii) then selecting equations to solve. This can be thought of as a *sequential approach* to problem-solving, and it guided a lot of how we taught and graded problem-solving. Good problem-solvers knew where to start, how to proceed, and how to show all the steps in arriving at the destination.

- In the new system, good problem-solvers should know how to make use of the various representations, and how to draw connections within and among them to discover new relationships. The point of "problem-solving" is more about building up the set of representations that describe the situation.

As students progress through the semester, the range and types of representations necessarily expands. Here is an example of students doing the work of organizing representations prior to solving for unknowns in an elevator problem.

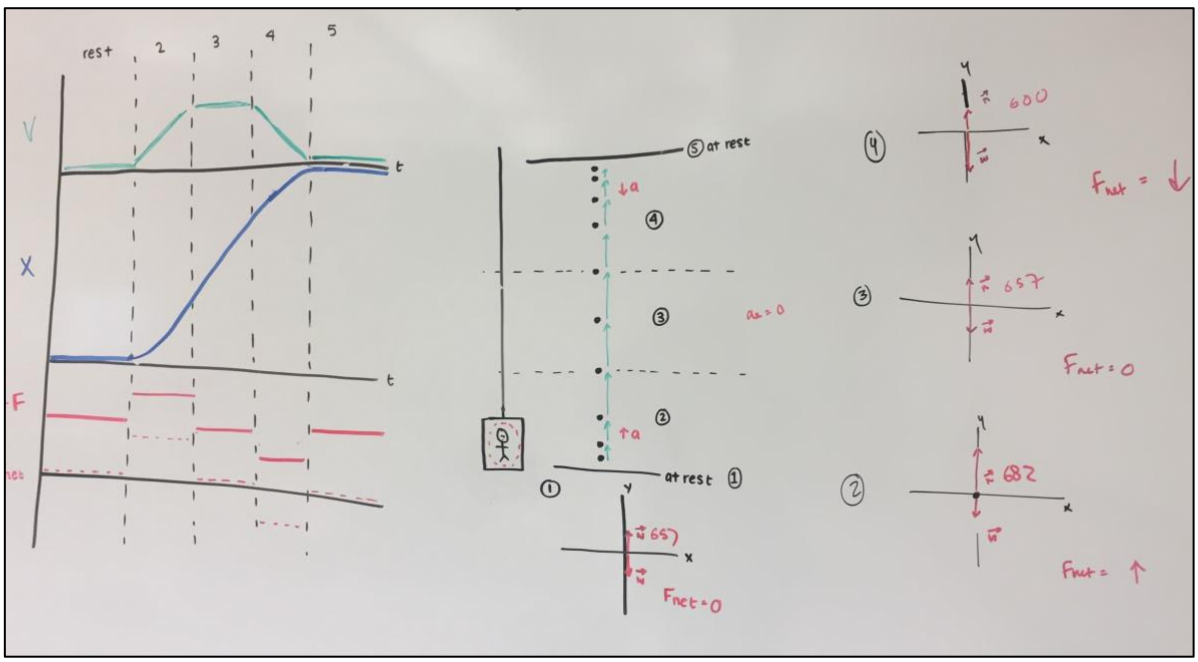


Figure: Multiple Representations of an Elevator Ascent

For a broader perspective, even in second semester physics, multiple representations are prominent in how we ask students to approach solving problems. Below a student uses multiple representations to predict all the current, voltage changes, and power outputs for a resistive circuit that was set up in lab.

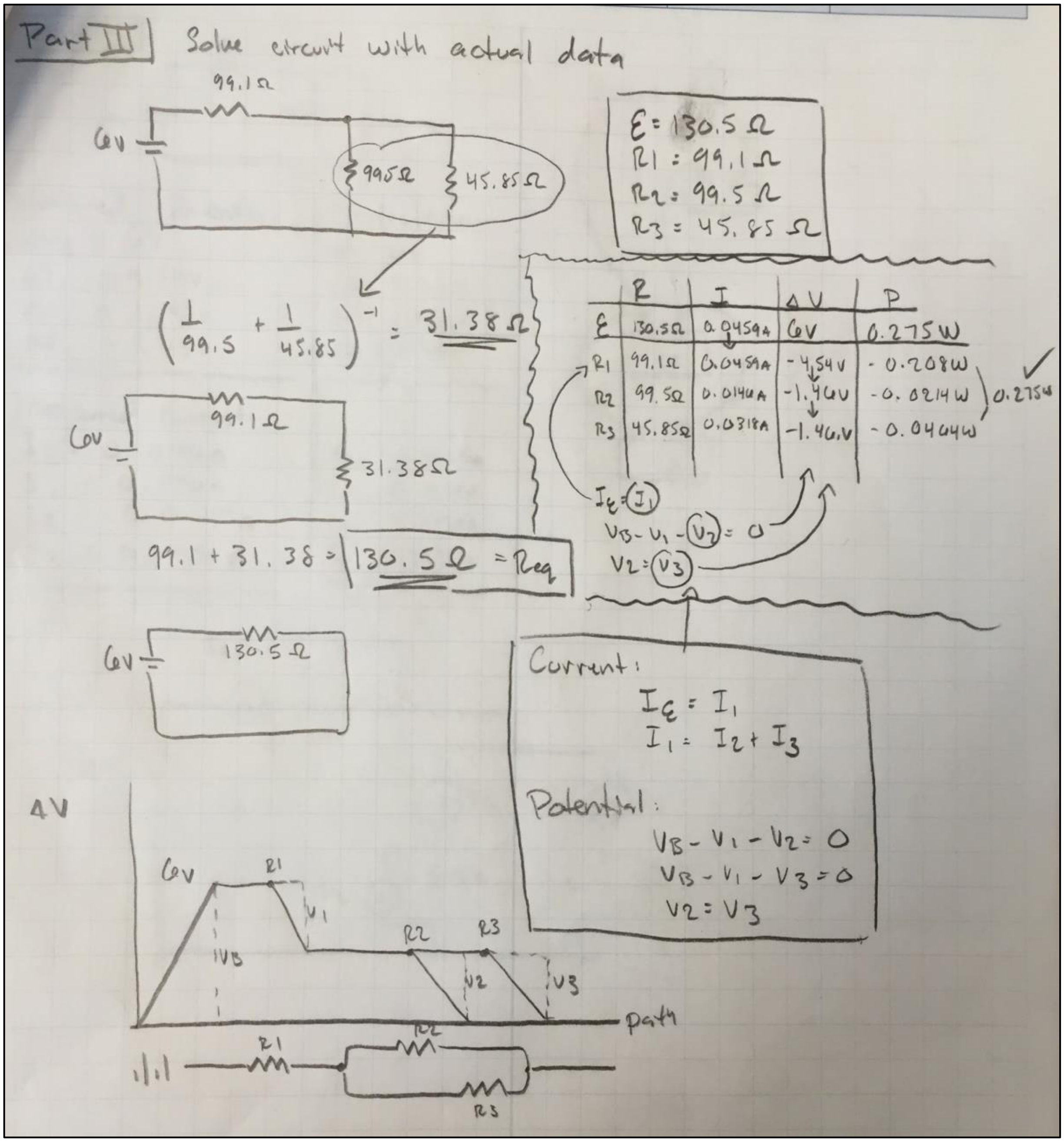


Figure: Multiple Representations of a Circuit

Some Nuts and Bolts

How do we get students to do this?

Lead by Example:

If we expect students to take this approach, we need to model it. Thus, this approach needs to be used when we as instructors model how to solve problems as well. Here is an example from early in the semester, of me modeling the use of multiple representations for a uniform motion problem.

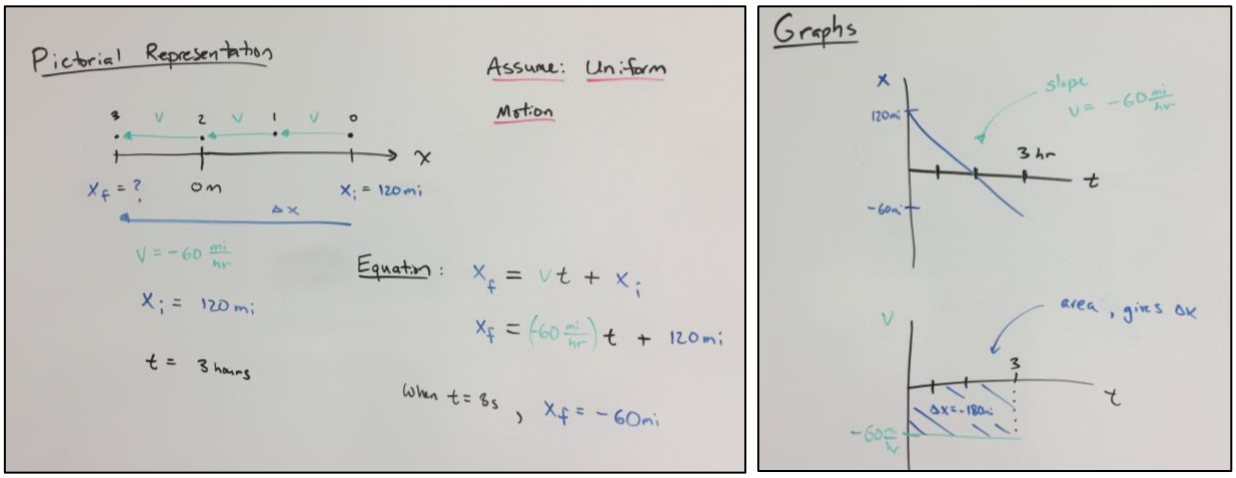


Figure: Instructors Model Use of Multiple Representations Early On

Use Must Haves:

One way the curriculum holds students accountable to doing this work is through the use of "must haves" that are associated with problems they solve. "Must Haves" are a list of the representations that must be included. An example for the elevator problem is shown below.

Early in the semester, you will likely be telling students exactly what is required. As time goes on, however, you might try getting students to help decide what should be on the "must have" lists. This helps them to start thinking about what tools will be most useful for particular problems, and what they would want to see to understand their peers' work.

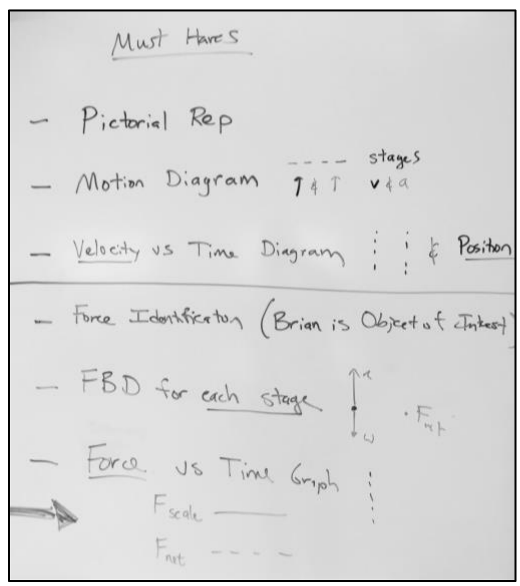


Figure: A List of Required Representations

Dealing with Equations:

Another tip relates to equations as a representation. I've learned through teaching this curriculum that a rule like, "You can't use an equation without also showing a graph (or other representation) that illustrates it" is helpful. I encourage students to enter their equation into a graphing calculator or online calculator (like Desmos) to verify that their equation matches their expectation for graph, especially before using the equation to solve for anything.

Here is an example of me modeling this in a projectile motion example problem:

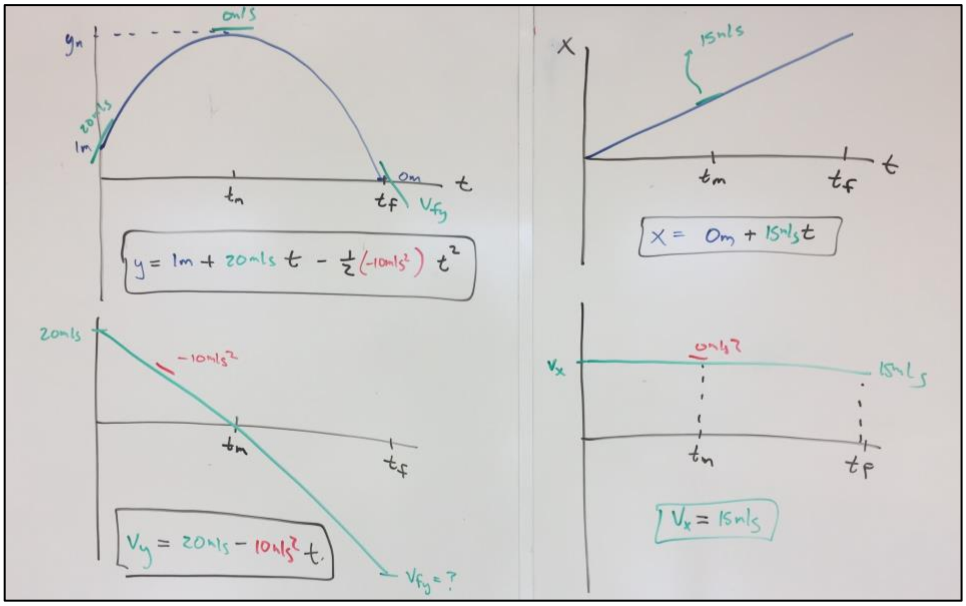
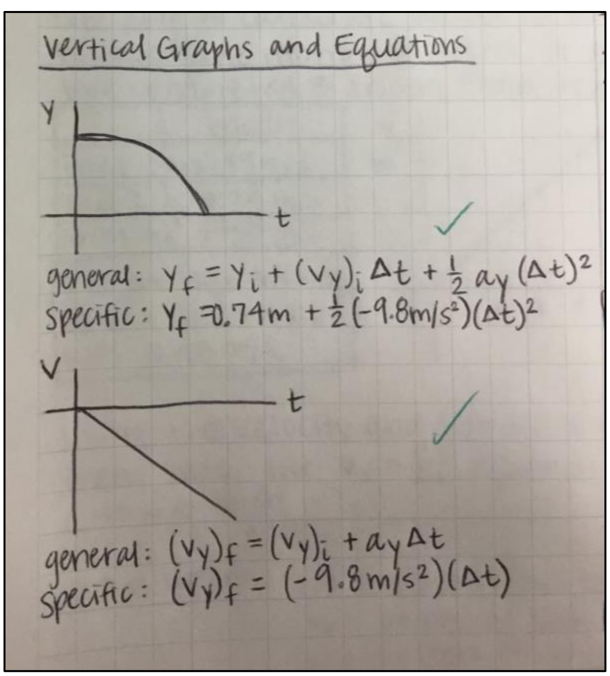
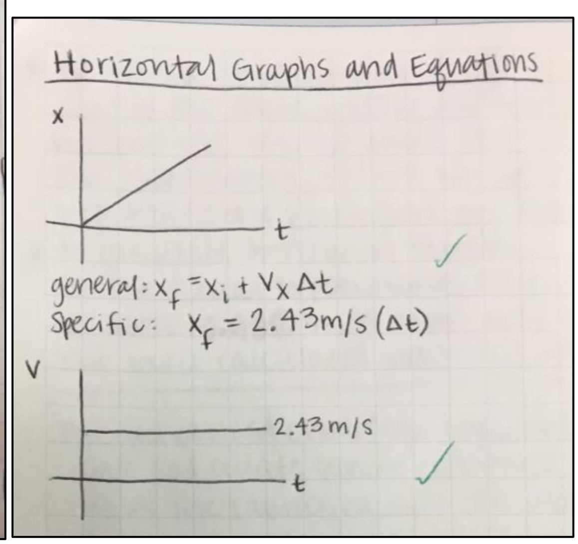


Figure: An Example of Linking Equations to Graphs

And an example of students doing similarly for a lab write-up.

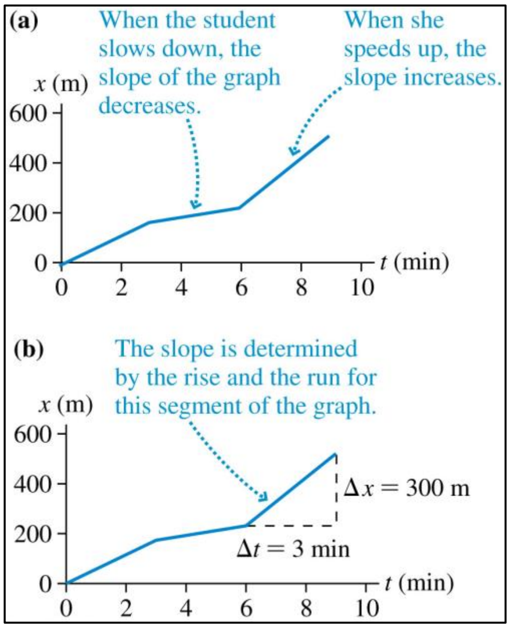




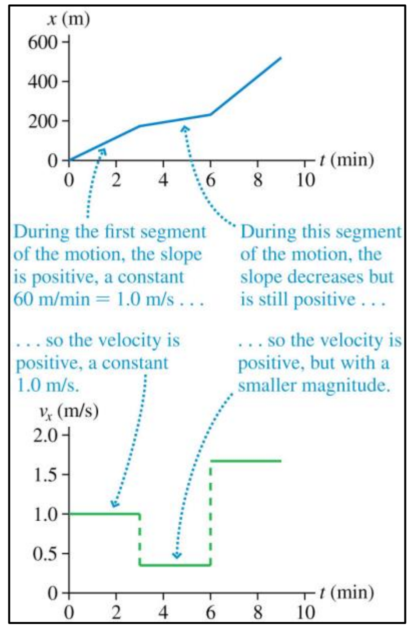
Two Kinds of Skill Practice: Within and Across Representation

In order to prepare students for problem-solving using multiple representations, there are two different kinds of skill practice we provide:

- Working within a single representation
- Working across different representations



Understanding a Single Representation

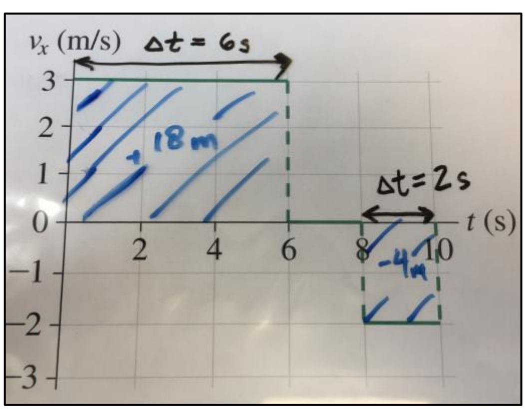


Connecting Two Representations

(Both figure from Knight's "College Physics" Text that students read)

<u>Example of In-class Practice within Representations:</u>

For example, in class, students might work a "mini whiteboard problem" to practice with skill of finding the area under velocity vs. time curve to calculate displacement, or discuss a clicker question about finding velocity from a position vs. time graph.



Here is a position graph of an object:

At *t* = 1.5 s, the object's velocity is

A. 40 m/s
B. 20 m/s
C. 10 m/s
D. 6.6 m/s
E. None of the above

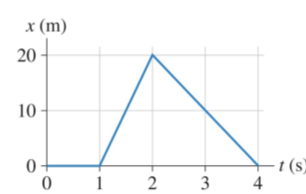


Figure: Examples of Working within a Single Representation (whiteboard and clicker question)

Examples of In-Class Practice Across Representations

To practice reasoning across representations, students might each student be given a single representation, and then be tasked with finding the other people in the room that match theirs. In a different classroom activity, students work a computer exercise to decide which representation doesn't belong the set.

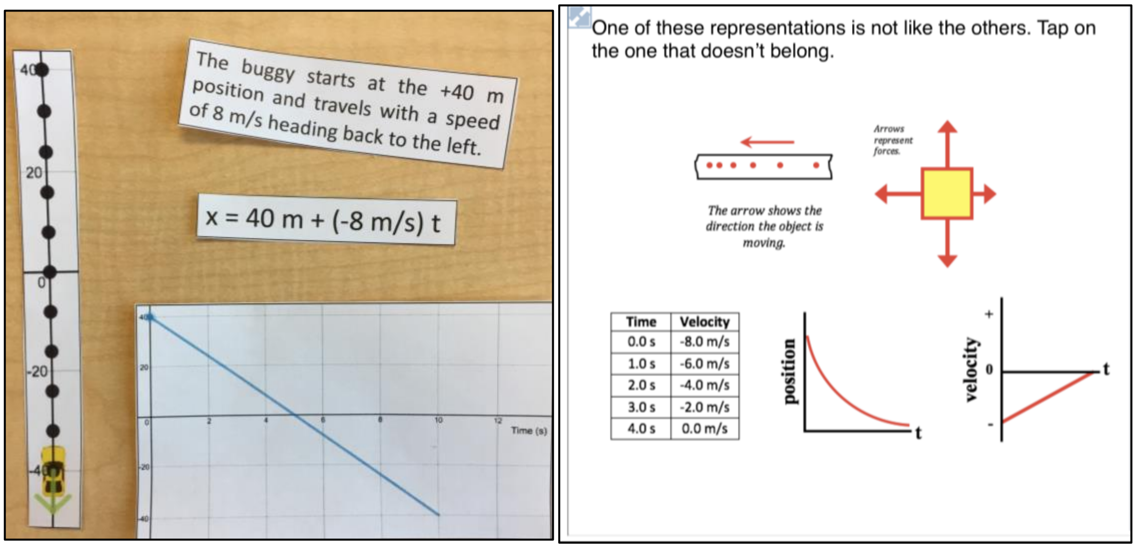


Figure: Examples of Working Across Representations
(computer exercise from the physics classroom.)

Ending Commentary

This brief introduction to solving problems with multiple representations is incomplete and probably leaves you with lots of questions, concerns, etc. It certainly takes some time (and practice) to get comfortable working problems in this manner but also in being able to quickly "assess" student work. I think one of the biggest challenges instructors will likely face is having to develop a modified sense of what a 'sophisticated' or 'rigorous' solution from students should look like.

Following in this series is more information about what the teaching of problem-solving actually looks like, whereas this focused on what the products of problem-solving might look like.